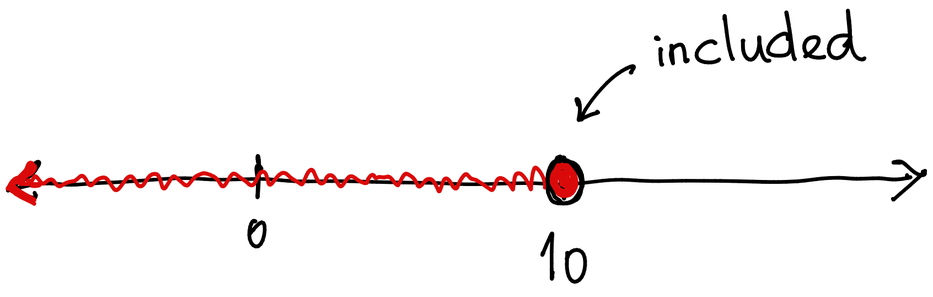
Class 4 - §1.7 Inequalities & §1.8 Absolute Value

How to represent an <u>interval</u>:



Line notation

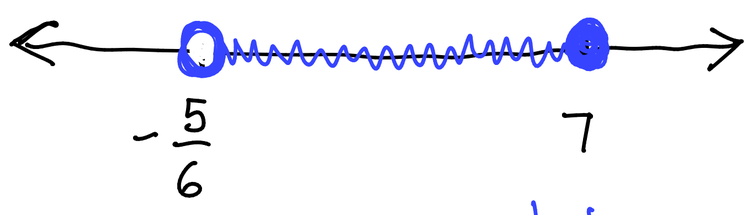
$(-\infty, 10]$ — included; "not included"; closed interval

interval notation

$\{x \mid x \leq 10\}$ or $\{x \in \mathbb{R} \mid x \leq 10\}$

set notation

<u>Ex</u>: Represent $(-\frac{5}{6}, 7]$ on line & set notation (Finite interval)



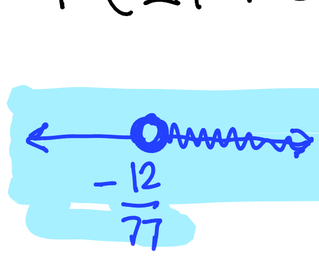
line notation

$\{x \mid -\frac{5}{6} < x \leq 7\}$ → half open interval

set notation

Regular inequalities:

<u>Ex</u>: $12(6-5y) + 3y < 4(21+5y)$. <u>Represent the solution in all the ways possible.</u>



or $(-\frac{12}{77}, \infty)$ or $\{y \mid y > -\frac{12}{77}\}$

$12(6-5y) + 3y < 4(21+5y)$

$72 - 60y + 3y < 84 + 20y$

$72 - 57y < 84 + 20y$

$-20y \quad -20y$

$72 - 77y < 84$

$-72 \quad -72$

$-77y < 12$

$\frac{-77y}{-77} < \frac{12}{-77}$

$y > -\frac{12}{77}$

$72 - 57y < 84 + 20y$

$-84 + 57y \quad -84 + 57y$

$-\frac{12}{77} < \frac{77y}{77} \iff -\frac{12}{77} < y$

Ex: $\frac{30-17w}{6} - \frac{w}{6} < -18$. Represent the answer in 3 ways.

$6\left(\frac{30-17w}{6} - \frac{w}{6}\right) < (-18)6$

$30 - 17w - w < -78$

$30 - 18w < -78$

$-30 \qquad -30$

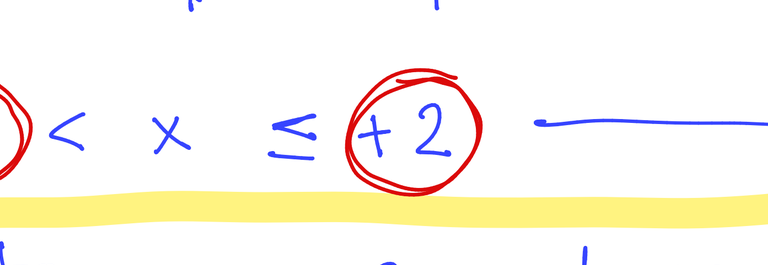
$\frac{-18w}{-18} < \frac{-108}{-18}$

$w > 6$

$\frac{-108}{-18} \begin{matrix}\div 2\\ \div 2\end{matrix} = \frac{54}{9} \begin{matrix}\div 9\\ \div 9\end{matrix} = 6$

$\{w \mid w > 6\}$ set

$(6, \infty)$ interval



line

Solve Three-part inequality

Recall: $-\frac{5}{6} < x \leq 7$ is a three-part inequality.

Ex: $3 < 4x + 7 \leq 15$

$-7 \qquad -7 \qquad -7$

$\Leftrightarrow \frac{-4}{4} < \frac{4x}{4} \leq \frac{+8}{4}$

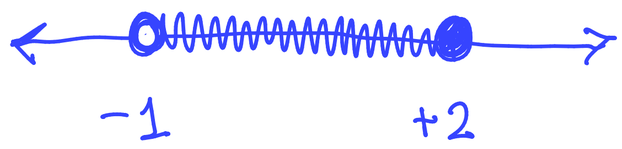
$\Leftrightarrow -1 < x \leq +2$

$\{x \mid -1 < x \leq -2\}$ → represents $x \leq -2$ and $x > -1$

-1 is greater than -2

represents $x \leq -2$ or $x > -1$

This inequality has no solution



$(-1, 2]$

Ex: $^{3}(-9) < \left(\frac{8-7x}{3}\right)^{3} < (-2)^{3}$

$-9 < \frac{8-7x}{3} < -2$

$\Leftrightarrow -27 < 8-7x < -6$

$-8 \quad -8 \quad -8$

$\Leftrightarrow \frac{-35}{-7} < \frac{-7x}{-7} < \frac{-14}{-7}$

$\Leftrightarrow \boxed{5 > x > 2}$

↓ REWRITE

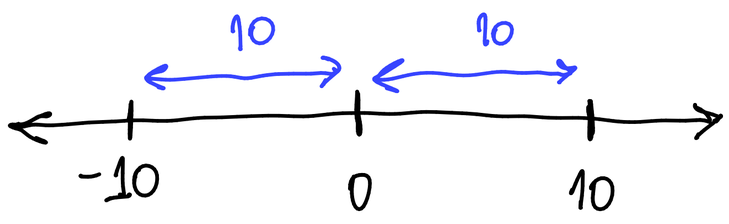
$\boxed{2 < x < 5}$

set: $\{x \mid 2 < x < 5\}$

line:

interval: $(2,5)$

Absolute Value



What is the distance from −10 to 0?

is 10. We represent this as $|-10| = 10$.

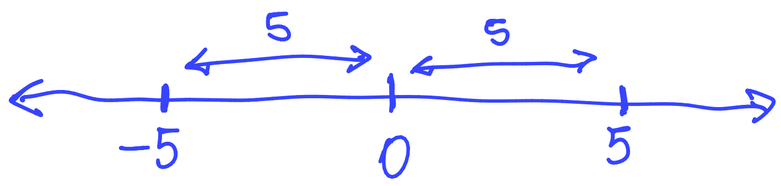
What is the distance from 10 to 0?

is 10. We represent this as $|10| = 10$.

$|0| = 0$

(mathematically)

How can we <u>represent</u> "all numbers with distance 5 from 0"?



$|x| = 5 \Leftrightarrow x = -5$ or $x = 5$

<u>Ex1</u>: $|x| = 7 \Leftrightarrow x = 7$ or $\boxed{x = -7}$

<u>Ex2</u>: $|x+7| = 9 \Leftrightarrow x+7 = 9$ or $x+7 = -9$

$-7 \quad -7 \qquad -7 \quad -7$

$\Leftrightarrow \boxed{x = 2}$ or $\boxed{x = -16}$

<u>Ex3</u>: $|3x+10| = -1000$, NO SOLUTION since $-1000 < 0$.

Ex 4: $|8x-1|+8=19$

$-8 \quad -8$

$\Leftrightarrow |8x-1| = 11$ → $8x-1$ has distance 11 from 0.

$\Leftrightarrow 8x-1 = -11$ or $8x-1 = 11$

$+1 \quad +1 \qquad +1 \quad +1$

$\Leftrightarrow \frac{8x}{8} = \frac{-10}{8}$ or $\frac{8x}{8} = \frac{12}{8}$

$\Leftrightarrow \boxed{x = -\frac{5}{4}}$ or $\boxed{x = \frac{3}{2}}$

Solution set $\left\{-\frac{5}{4}, \frac{3}{2}\right\}$

Ex 5: $|4x+20| = 0$ → $4x+20$ has dist. 0 from 0.

$4x+20=0 \Leftrightarrow 4x=-20 \Leftrightarrow \boxed{x=-5}$

$-20 \quad -20 \qquad \frac{4x}{4} = \frac{-20}{4}$

Ex 6: $3|9x-20|-10=11$

$3|9\cdot 7-20|-10 = 3|63-20|-10$

$= 3|43|-10$

$= 3\cdot 43 - 10$

$= 126-10$

$= 116$

$3|9\cdot\frac{13}{9}-20|-10 \stackrel{?}{=} 11$

$3|13-20|-10 \stackrel{?}{=} 11$

$3|7|-10 \stackrel{?}{=} 11$

$3\cdot 7 - 10 \stackrel{?}{=} 11$

$21-10=11$ ✓

$3|9x-20|-10=11$

$+10 \quad +10$

$\frac{3|9x-20|}{3} = \frac{21}{3} \Rightarrow$

$|9x-20| = 7$

$9x-20=+7$ or $9x-20=-7$

$+20 \quad +20 \qquad +20 \quad +20$

$\frac{9x}{9} = \frac{27}{9} \qquad \frac{9x}{9} = \frac{13}{9}$

$\boxed{x=3}$ $\boxed{x=\frac{13}{9}}$

Ex 7: Find all solutions for $|x^2 - 5x + 5| = 1$.

$x^2 - 5x + 5 = 1$ or $x^2 - 5x + 5 = -1$

$-1 \quad -1$ $+1 \quad +1$

$x^2 - 5x - 4 = 0$ or $x^2 - 5x + 6 = 0$

$(x-1)(x-4) = 0$ or $(x-2)(x-3) = 0$

$x = 1, x = 4$ or $x = 2, 3$

Solution set : $\{1, 2, 3, 4\}$.